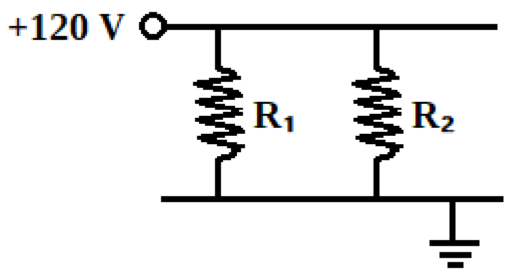
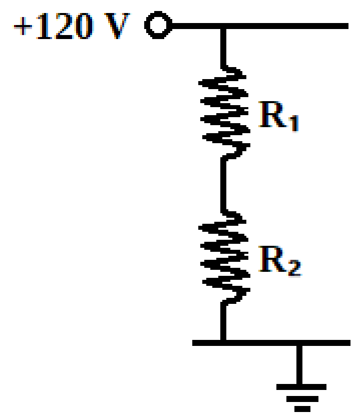
Q: Two resistors when connected in series to a 120-V line use one-fourth the power that is used when they are connected in parallel. If one resistor is 4.8 kΩ, what is the resistance of the other?

A:



Since the two resistors are in parallel, both are exposed to the same voltage, V. Therefore,

$$\begin{aligned} P_{\text{parallel}} &= \frac{V^2}{R_1} + \frac{V^2}{R_2} \\ &= V^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\ &= V^2 \left(\frac{R_1 + R_2}{R_1 R_2} \right) \end{aligned}$$



Since the two resistors are in series, they are both exposed to the same current. Therefore,

$$\begin{aligned} P_{\text{series}} &= I^2 R_1 + I^2 R_2 \\ &= I^2 (R_1 + R_2) \\ &= \left(\frac{V}{R_1 + R_2} \right)^2 (R_1 + R_2) \\ &= V^2 \left(\frac{1}{R_1 + R_2} \right) \end{aligned}$$

$$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{V^2 \left(\frac{R_1 + R_2}{R_1 R_2} \right)}{V^2 \left(\frac{1}{R_1 + R_2} \right)}$$

$$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{(R_1 + R_2)^2}{R_1 R_2}$$

Substituting in our numbers

$$4 = \frac{(4.8\,\text{k}\Omega + R_2)^2}{(4.8\,\text{k}\Omega) R_2}$$

and solving the resulting quadratic equation for R_2 yields

$$\boxed{R_2 = 4.8\,\text{k}\Omega}$$

(Although the resistors turn out to be of equal resistance in this case, this is not true in general. For instance, if the power ratio had been 5 instead of 4, then the second resistor could have been either $1.8\,\text{k}\Omega$ or $12.6\,\text{k}\Omega$.)